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Abstract

The classical binary error correcting codes, and subspace codes for error correction in random network coding are two different forms of error control coding. We identify common features between these two forms and study the relations between them using the aid of lattices. Lattices are partial ordered sets where every pair of elements has a least upper bound and a greatest lower bound in the lattice.

We shall demonstrate that many questions that connect these forms have a natural motivation from the viewpoint of lattices. We shall show that a lattice framework captures the notion of Singleton bound where the bound is on the size of the code as a function of its parameters. For the most part, we consider a special type of a lattice which has the geometric modular property. We will use a lattice framework to combine the two different forms. And then, in order to demonstrate the utility of this binding view, we shall derive a general version of Singleton bound. We will note that the Singleton bounds behave differently in certain respects because the binary coding framework is associated with a lattice that is distributive. We shall demonstrate that lack of distributivity gives rise to a weaker bound.

We show that Singleton bound for classical binary codes, subspace codes, rank metric codes and Ferrers diagram rank metric codes can be derived using a common technique. In the literature, Singleton bounds are derived for Ferrers diagram rank metric codes where the rank metric codes are linear. We introduce a generalized version of Ferrers diagram rank metric codes and obtain a Singleton bound for this version.

Next, we shall prove a conjecture concerning the constraints of embedding a binary coding framework into a subspace framework. We shall prove a conjecture by Braun, Etzion and Vardy, which states that any such embedding which contains

the full space in its range is constrained to have a particular size. Our proof will use a theorem due to Lovasz, a subspace counting theorem for geometric modular lattices, to prove the conjecture. We shall further demonstrate that any code that achieves the conjectured size must be of a particular type. This particular type turns out to be a natural distributive sub-lattice of a given geometric modular lattice.